Placement Exam for MA129

September 5, 2025

Riemann Sums

10 points for scoring at least 2 out of 4.

[A] Which of the following represents a right-hand Riemann approximation for the area under the curve $f(x) = x^2$ between $x = 0$ and $x = 5$ using 3 rectangles?

(i) $\sum_{n=1}^{3} n^2$

(ii) $\sum_{n=0}^{5} \frac{5}{3} n^2$

(iii) $\sum_{n=0}^{3} \frac{5}{3} \left(\frac{5n}{3}\right)^2$

(iv) $\sum_{n=0}^{2} \frac{5}{3} \left(\frac{5n}{3}\right)^2$

(v) $\sum_{n=1}^{3} \frac{5}{3} \left(\frac{5n}{3}\right)^2$

[B] Suppose $f(x)$ is a decreasing function on $[a, b]$. A midpoint Riemann approximation of $\int_a^b f(x)\,dx$ will be an

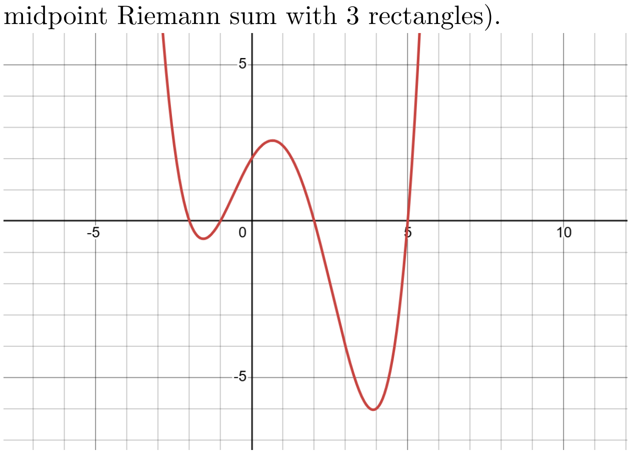
(i) Over-approximation

(ii) Under-approximation

(iii) Exact answer

(iv) Not enough information

[C] Below is the graph of a function $g(x)$. Use this graph to approximate M_3 on the interval $[0, 6]$ (the midpoint Riemann sum with 3 rectangles).



(i) 13

(ii) -15

(iii) -3

(iv) 0

[D] True or False: In a general Riemann sum, rectangles must all have the same width.

(i) TRUE

(ii) FALSE

Definition of the Definite Integral

18 points for scoring at least 3 out of 4.

[A] Which of the following is the definition of $\int_2^4 x\,dx$?

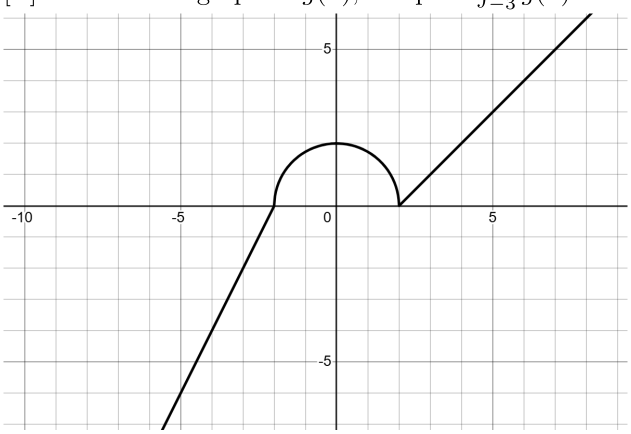
(i) $\frac{x^2}{2} + c$

(ii) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left(2 + \frac{2i}{n}\right)$

(iii) $\frac{4^2}{2} - \frac{2^2}{2}$

(iv) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}(i)$

[B] Below is the graph of $g(x)$, compute $\int_{-3}^{5} g(x)\,dx$



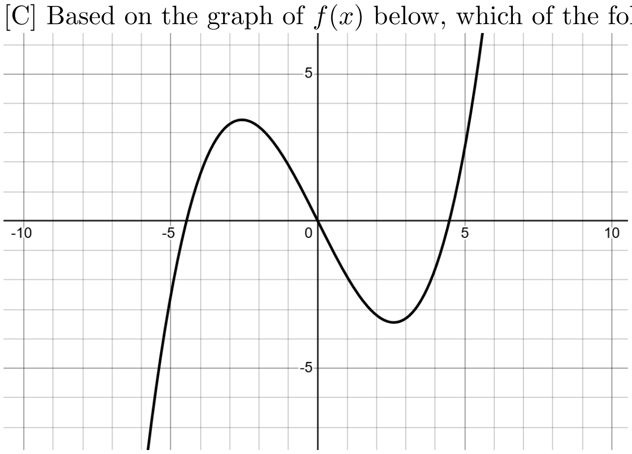
(i) $\frac{11}{2} + 2\pi$

(ii) $\frac{7}{2} + 4\pi$

(iii) $\frac{7}{2} + 2\pi$

(iv) $11 + 2\pi$

[C] Based on the graph of $f(x)$ below, which of the following integrals returns the largest number?



(i) $\displaystyle\int_{-2}^{0} f(x)\,dx$

(ii) $\int_{-2}^{2} f(x)\,dx$

(iii) $\int_{-4}^{2} f(x)\,dx$

(iv) $\int_{-4}^{0} f(x)\,dx$

(v) $\int_{-4}^{4} f(x)\,dx$

[D] Suppose we know $\int_{0}^{1} f(x)\,dx = -2$ and $\int_{1}^{5} f(x)\,dx = 6$. What is $\int_{5}^{0} 7f(x)\,dx$?

(i) -8

(ii) -4

(iii) 28

(iv) -28

(v) 56

Fundamental Theorem of Calculus

20 points for scoring at least 3 out of 4.

[A] Compute the derivative of the following function: $F(x) = \int_x^2 e^{t^3+2t+1}\,dt$

(i) e^{x^3+2x+1}

(ii) $-e^{x^3+2x+1}$

(iii) $(3x^2+2)e^{x^3+2x+1}$

(iv) $-(3x^2+2)e^{x^3+2x+1}$

[B] Evaluate the following definite integral: $\int_1^3 \frac{2}{x} - x^4 + 4\,dx$

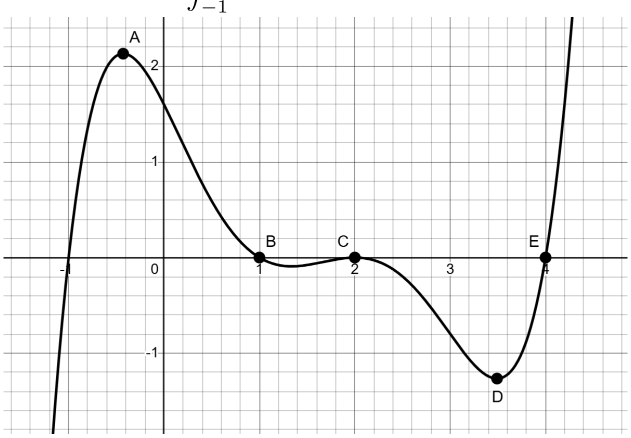
(i) $2\ln(3) - \frac{242}{5}$

(ii) $-\frac{2}{9} - 102$

(iii) $2\ln(3) - \frac{242}{5} + 8$

(iv) $-\frac{242}{5} + 10 - \frac{2}{9}$

[C] Let $G(x) = \int_{-1}^x g(t)\,dt$ where $g(t)$ is shown below. $G(x)$ has a local minimum at what x value(s)?



(i) A

(ii) B

(iii) C

(iv) D

(v) E

[D] Evaluate the definite integral $\int_0^4 f(x)\,dx$ where

$$f(x) = \begin{cases} 3+5x & \text{if } x < -2 \\ \frac{2}{x+3} & \text{if } -2 \le x \le 3 \\ \sqrt{x} & \text{if } x > 3 \end{cases}$$

(i) $\dfrac{28}{3} + 2\ln(6) - 2\sqrt{3}$

(ii) $\dfrac{16}{3} + \ln(6) - 2\sqrt{3}$

(iii) $\dfrac{28}{3} + 2\ln(6)$

(iv) 52

Integration Techniques

25 points for scoring at least 3 out of 5.

[A] Evaluate the following integral: $\displaystyle\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx$

(i) $\frac{1}{2}\sin(\sqrt{x}) + c$

(ii) $2\sin(\sqrt{x}) + c$

(iii) $\sin(\sqrt{x}) + c$

(iv) $\frac{\sin(\sqrt{x})}{\frac{2}{3}x^{3/2}} + c$

(v) $\sin(x) + c$

[B] Evaluate the following integral: $\displaystyle\int t(\ln t)^2\,dt$

(i) $\frac{t^2}{2}(\ln t)^2 - \frac{t^2}{2}\ln t - \frac{t^2}{4} + c$

(ii) $\frac{t^2}{2}(\ln t)^2 - \frac{t^3}{3}\ln t + \frac{t^3}{9} + c$

(iii) $\frac{t^2}{2}(\ln t)^2 - \frac{t^2}{2}\ln t + \frac{t^2}{4} + c$

(iv) $\frac{(\ln t)^3}{3} + c$

[C] Evaluate the following integral: $\displaystyle\int \cot(\theta)\csc^5(\theta)\,d\theta$

(i) $-\frac{\csc^5\theta}{5} + c$

(ii) $\frac{\csc^5\theta}{5} + c$

(iii) $-\frac{\csc^6\theta\cot^{\theta}}{10} + c$

(iv) $\csc^4\theta + c$

[D] Evaluate the following integral: $\displaystyle\int \frac{x^2}{\sqrt{x^2-9}}\,dx$

(i) $3\sqrt{x^2-9} + c$

(ii) $3\sec^{-1}(x/3) - \frac{9}{\sqrt{x^2-9}} + c$

(iii) $3\ln\left|\frac{\sqrt{x^2-9}}{3}\right| + c$

(iv) $3\ln\left|\frac{x}{3}\cdot\frac{\sqrt{x^2-9}}{3}\right| - \frac{\sqrt{x^2-9}}{x} + c$

[E] Evaluate the following integral: $\displaystyle\int \frac{3}{2x^2+x}\,dx$

(i) $3\ln|x| - 6\ln|2x+1| + c$

(ii) $-6\ln|x| + 3\ln|2x+1| + c$

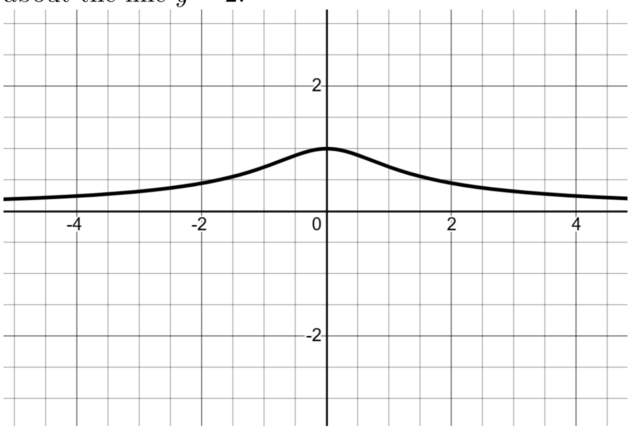
(iii) $-6\ln|x| + \frac{3}{2}\ln|2x+1| + c$

(iv) $3\ln|x| - 3\ln|2x+1| + c$

Volumes of Solids of Revolutions

15 points for scoring at least 2 out of 3.

[A] Which of the following represent the correct antiderivative of the function used to find the volume of the solid obtained by rotating the region bounded by $y = 0$, $y = \dfrac{1}{\sqrt{x^2+1}}$ (shown below), $x = -2$, $x = 2$ about the line $y = 2$.



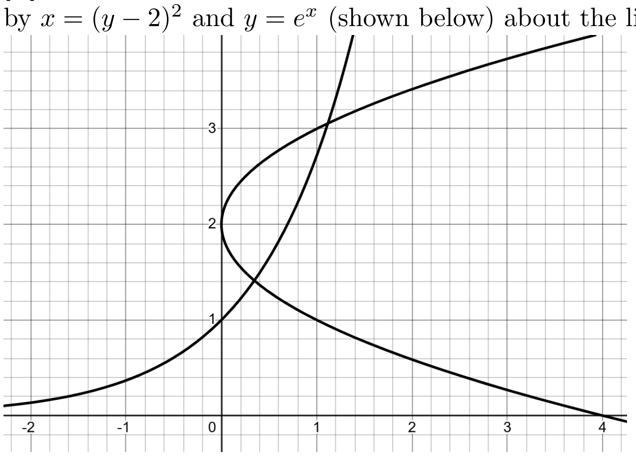
(i) $\pi(4x + \tan^{-1}(x))$

(ii) $\pi(2x - \tan^{-1}(x))$

(iii) $\pi(2x + \tan^{-1}(x))$

(iv) $\pi(\tan^{-1}(x))$

[B] Which of the following integrals represent the volume of the solid obtained by rotating the region bounded by $x = (y-2)^2$ and $y = e^x$ (shown below) about the line $x = 0$?



(i) $\pi \displaystyle\int_{1.41}^{3.057} (\ln(y))^2 - (y-2))^4 \; dy$

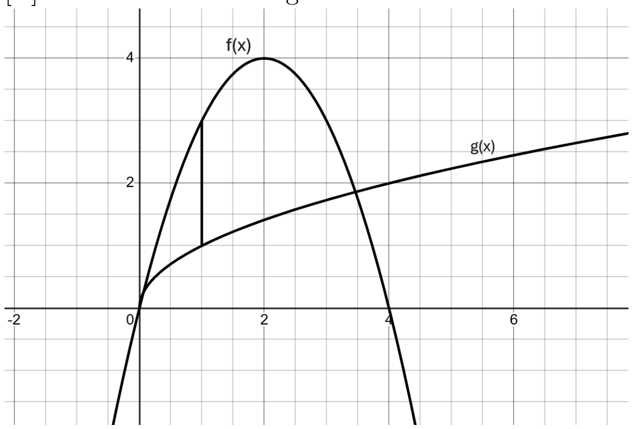
(ii) $2\pi \displaystyle\int_{.345}^{1.117} y\left(\ln(y) - (y-2)^2\right) \; dy$

(iii) $\pi \displaystyle\int_{.345}^{1.117} \left(\sqrt{x}+2\right)^2 - (e^x)^2 \; dx$

(iv) $2\pi \displaystyle\int_{1.41}^{3.057} y\left(\ln(y) - (y-2)^2\right) \; dy$

(v) $\pi \int_{.345}^{1.117} \left(\sqrt{x} + 2 - e^{x}\right)^2 \, dx$

[C] The result of rotating the vertical line shown below about the line $y = 4$ is a



(i) A cylindrical shell with height $f(x) - g(x)$ and radius x

(ii) A washer with outer radius $4 - f(x)$ and inner radius $4 - g(x)$

(iii) A washer with outer radius $f(x) + 4$ and inner radius $g(x) + 4$

(iv) A cylindrical shell with height $f(x) - g(x)$ and radius $x - 4$

(v) A washer with with outer radius $4 - g(x)$ and inner radius $4 - f(x)$

Other Applications of the Definite Integral

6 points for scoring at least 1 out of 3.

[A] Find the volume of the solid whose base is a triangle with vertices $(0,0),(-6,3),(3,3)$ and whose cross-sections perpendicular to the y-axis are squares.

(i) $\frac{27}{4}$

(ii) $\frac{243}{12}$

(iii) $\frac{243}{12}+162$

(iv) $\frac{27}{4}-27$

[B] Calculate the arc-length of $y=x^{3/2}$ on the interval $[1,2]$

(i) 4.693

(ii) 1.863

(iii) 1.351

(iv) 2.086

[C] Determine if $\displaystyle\int_0^\infty xe^{-x^2}$ converges, and if so, evaluate it.

(i) $\frac{1}{2}$

(ii) 2

(iii) 0

(iv) $-frac12$

(v) -2

(vi) Diverges

Differential Equations

6 points for scoring at least 2 out of 2.

[A] Find the solution to the differential equation $\sqrt{1-x^2}y' = xy$ that satisfies $y(0) = 1$.

(i) $e^{\sqrt{1+x^2}} - 1$

(ii) $e^{\sqrt{1+x^2}-1}$

(iii) $e^{\sqrt{1+x^2}} - 4$

(iv) $e^{\sqrt{1+x^2}-4}$

(v) $e^{\sqrt{1+x^2}} - 2$

(vi) $e^{\sqrt{1+x^2}-2}$

[B] Which of the n-values below make $y = x^n$ a solution to $y'' - \dfrac{12y}{x^2} = 0$?

(i) 2

(ii) 3

(iii) 4

(iv) None of the above