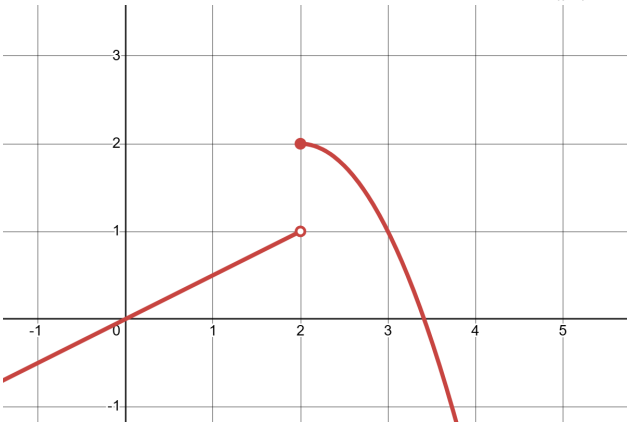
Placement Exam for MA126

September 5, 2025

Understanding Limits Graphically

12 points for scoring at least 6 out of 8.

[A] Consider the graph of $f(x)$ below. What is $\lim_{x \to 2^-} f(x)$?



(i) 1

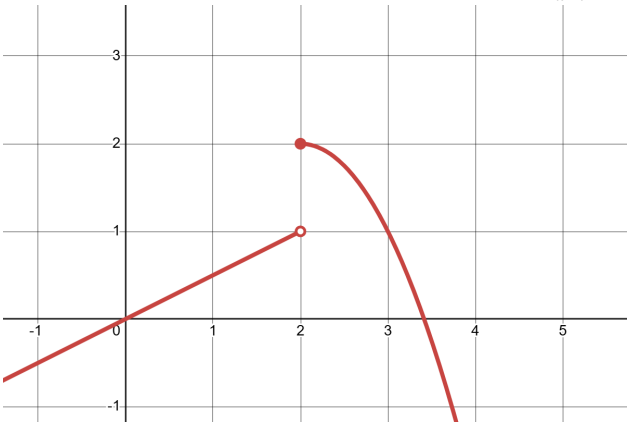
(ii) 1.5

(iii) 2

(iv) both 1 and 2

(v) does not exist

[B] Consider the graph of $f(x)$ below. What is $\lim_{x \to 2^+} f(x)$?



(i) 1

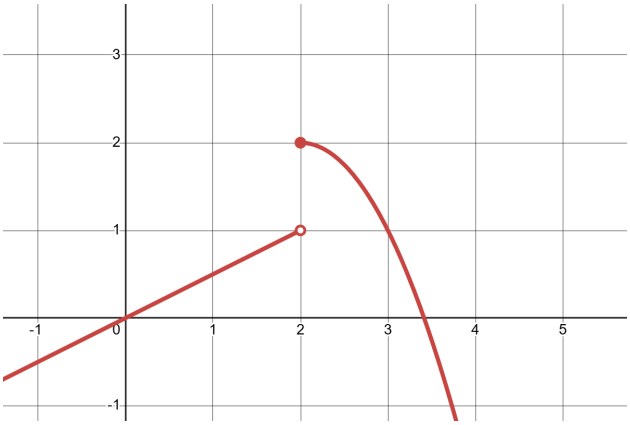
(ii) 1.5

(iii) 2

(iv) both 1 and 2

(v) does not exist

[C] Consider the graph of $f(x)$ below. What is $\lim_{x \to 2} f(x)$?



(i) 1

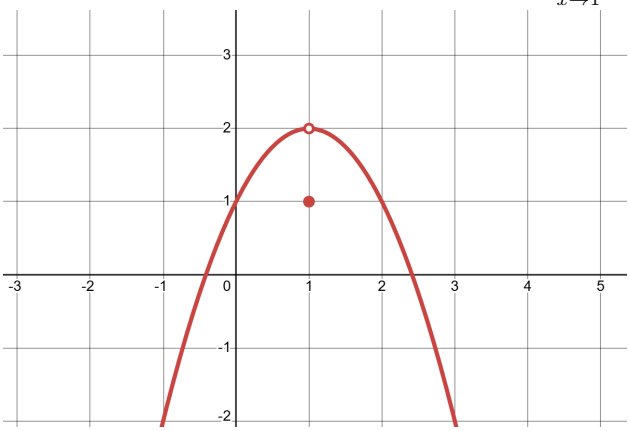
(ii) 1.5

(iii) 2

(iv) both 1 and 2

(v) does not exist

[D] Consider the graph of $f(x)$ below. What is $\lim_{x \to 1} f(x)$?



(i) 1

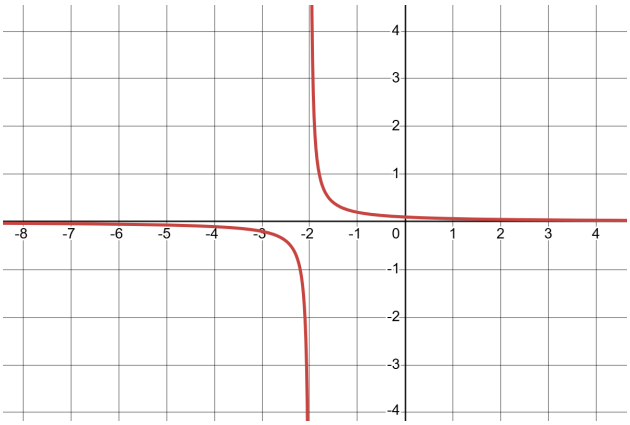
(ii) 1.5

(iii) 2

(iv) both 1 and 2

(v) does not exist

[E] Consider the graph of $f(x)$ below. What is $\lim_{x \to -2^-} f(x)$?



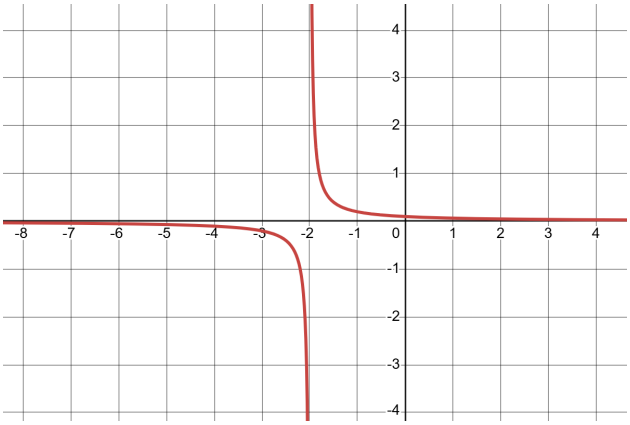
(i) $-\infty$

(ii) 0

(iii) $+\infty$

(iv) both $-\infty$ and $+\infty$

[F] Consider the graph of $f(x)$ below. What is $\lim_{x \to -2^+} f(x)$?



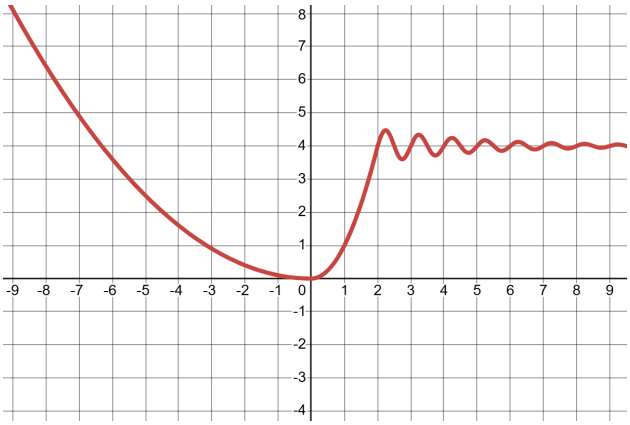
(i) $-\infty$

(ii) 0

(iii) $+\infty$

(iv) both $-\infty$ and $+\infty$

[G] Consider the graph of $f(x)$ below. What is $\lim_{x \to -\infty} f(x)$?



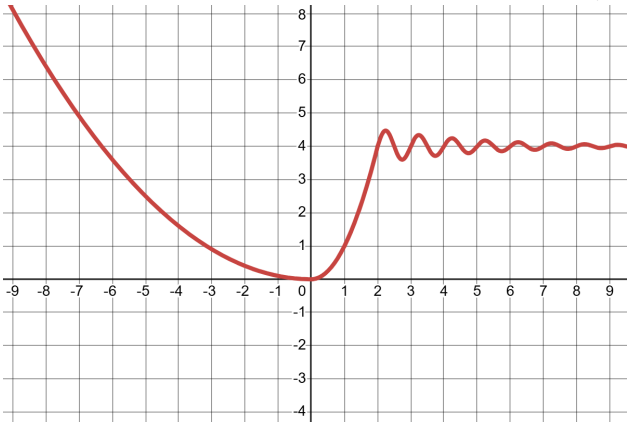
(i) $-\infty$

(ii) 0

(iii) 8

(iv) $+\infty$

[H] Consider the graph of $f(x)$ below. What is $\lim_{x \to +\infty} f(x)$?



(i) $-\infty$

(ii) 0

(iii) 4

(iv) $+\infty$

(v) does not exist

Computing Limits Algebraically

6 points for scoring at least 2 out of 3.

[A] Compute $\lim_{x \to 1} \dfrac{x^2 + x - 2}{x^2 - 4x + 3}$

(i) $-\frac{3}{2}$

(ii) -1

(iii) $-\frac{2}{3}$

(iv) 0

(v) 1

(vi) does not exist

[B] Compute $\lim_{x \to 1} \dfrac{\frac{1}{x} - 1}{x - 1}$

(i) -1

(ii) $-\frac{1}{2}$

(iii) 0

(iv) $\frac{1}{2}$

(v) 1

(vi) does not exist

[C] Compute $\lim_{x \to 1} \dfrac{\sqrt{x} - 1}{x - 1}$

(i) -1

(ii) $-\frac{1}{2}$

(iii) 0

(iv) $\frac{1}{2}$

(v) 1

(vi) does not exist

Definition and Meaning of the Derivative

22 points for scoring at least 3 out of 4.

[A] Which of these is the derivative of $f(x) = \sqrt{x+1}$ at $x = 3$?

(i) $\lim_{x\to 3} \dfrac{\sqrt{x+h+1}-2}{x-3}$

(ii) $\lim_{h\to 3} \dfrac{\sqrt{4+h}-2}{h}$

(iii) $\lim_{x\to 0} \dfrac{\sqrt{x+1}-2}{x-3}$

(iv) $\lim_{h\to 0} \dfrac{\sqrt{4+h}-2}{h}$

[B] Consider the table of values of $g(x)$ below.

x	0	0.5	1	1.5	2
$g(x)$	-1	1	1.3	2	4

Which of the following is the best approximation of $g'(1)$?

(i) 1.3

(ii) $\dfrac{1.3-1}{1-0.5}$

(iii) $\dfrac{1.3-(-1)}{1-0}$

(iv) $\dfrac{4-(-1)}{2-0}$

[C] Suppose a manufacturing plant has a profit function $P(x)$ where the output is profit measured in dollars given the input x is number of units produced. What is the meaning of "$P'(100) = -10.31$"?

(i) When 100 units are produced, the manufacturing plant has a deficit of \$10.31.

(ii) When 100 units are produced, profit is increasing at a rate of \$10.31 per unit produced.

(iii) When 100 units are produced, profit is decreasing at a rate of \$10.31 per unit produced.

(iv) When 100 units are produced, profit is increasing at a rate of \$10.31.

(v) When 100 units are produced, profit is decreasing at a rate of \$10.31.

[D] Suppose $f(x)$ is a function, a is a number, and the following limit exists:

$$L = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}.$$

Which of the following are true statements?

(I) The number L is the slope of the tangent line to $f(x)$ at $x = a$.

(II) The number L is the limit of the slopes of secant lines passing through $(a, f(a))$ and an arbitrary point $(x, f(x))$ as the point $(x, f(x))$ slides along the curve toward the point $(a, f(a))$.

(III) The curve $y = f(x)$ might have a vertical tangent line at $x = a$.

(IV) The curve $y = f(x)$ might have a horizontal tangent line at $x = a$.

(V) The function $f(x)$ is continuous at $x = a$

(i) (I) only

(ii) (I) and (II)

(iii) (I), (III), and (V)

(iv) (I), (IV), and (V)

(v) (I), (II), (IV), and (V)

(vi) (II), (III), (IV), and (V)

(vii) (I), (II), (III), (IV), and (V)

Derivative Rules

10 points for scoring at least 3 out of 4.

[A] What is the derivative of $\frac{1}{\sqrt[5]{x^3}}$?

(i) $-\frac{5}{3}x^{-8/3}$

(ii) $-\frac{3}{5}x^{-8/5}$

(iii) $\frac{3}{5}x^{-2/5}$

(iv) $-\frac{3}{5}x^{2/5}$

(v) $\frac{1}{\frac{3}{5}x^{-2/5}}$

(vi) $\frac{1}{\frac{5}{3}x^{2/3}}$

[B] Suppose $f(1) = 2$, $f'(1) = 3$, $g(1) = 4$, and $g'(1) = 5$. What is the derivative of $f(x)g(x)$ evaluated at $x = 1$?

(i) -14

(ii) -2

(iii) 2

(iv) 14

(v) 15

(vi) 22

(vii) 26

[C] Suppose $f(1) = 2$, $f'(1) = 3$, $g(1) = 4$, and $g'(1) = 5$. What is the derivative of $\frac{f(x)}{g(x)}$ evaluated at $x = 1$?

(i) $-\frac{11}{8}$

(ii) $-\frac{1}{2}$

(iii) $-\frac{1}{8}$

(iv) $\frac{1}{8}$

(v) $\frac{1}{2}$

(vi) $\frac{3}{5}$

(vii) $\frac{11}{8}$

[D] What is the derivative of $e^{4x^3} + \sin(2x)$?

(i) $e^{4x^3} + \cos(2x)$

(ii) $e^{4x^3} + 2\cos(2x)$

(iii) $e^{4x^3} - 2\cos(2x)$

(iv) $12x^2e^{4x^3} + \cos(2x)$

(v) $12x^2e^{4x^3} - \cos(2x)$

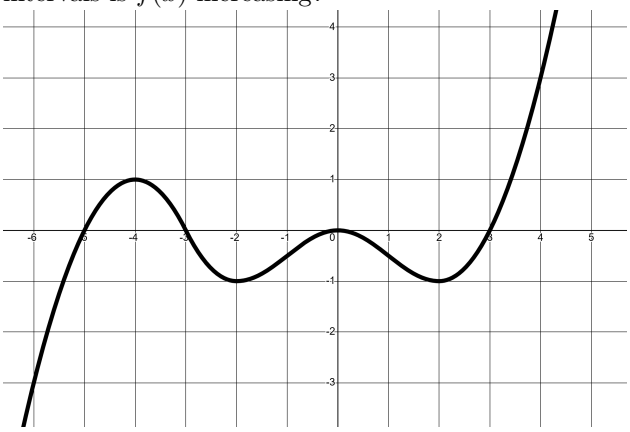
(vi) $12x^2e^{4x^3} + 2\cos(2x)$

(vii) $12x^2e^{4x^3} - 2\cos(2x)$

Derivatives and the Graph of a Function

16 points for scoring at least 3 out of 4.

[A] Consider the following graph of *the derivative* $f'(x)$ of a function $f(x)$. On which of the following intervals is $f(x)$ increasing?



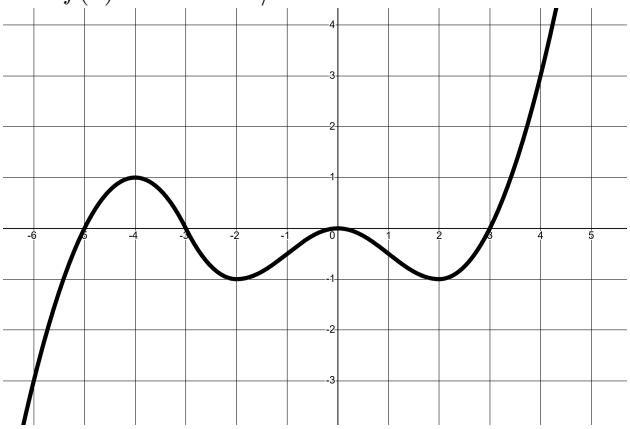
(i) $-5 < x < -3$

(ii) $-4 < x < -2$

(iii) $-3 < x < 0$

(iv) $2 < x < 4$

[B] Consider the following graph of *the derivative* $f'(x)$ of a function $f(x)$. For which of the following x-values does $f(x)$ have a local/relative minimum?



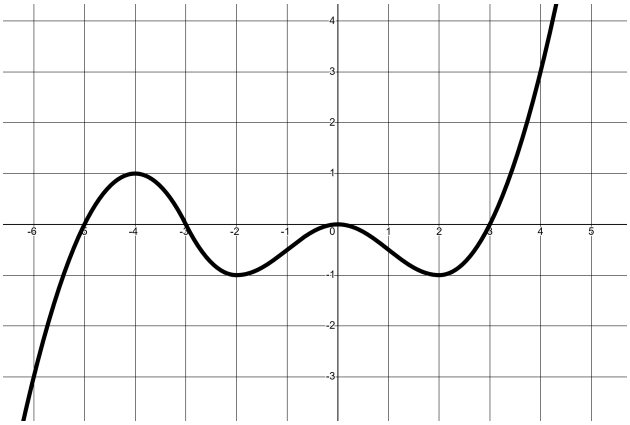
(i) $x = -3$

(ii) $x = 1$

(iii) $x = 2$

(iv) $x = 3$

[C] Consider the following graph of *the derivative* $f'(x)$ of a function $f(x)$. On which of the following intervals is $f(x)$ concave down?



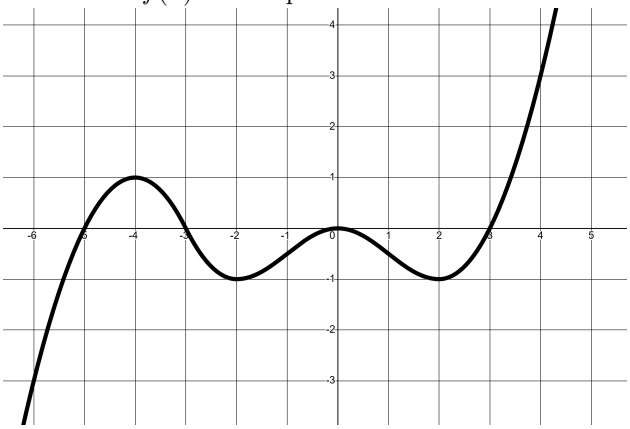
(i) $-5 < x < -3$

(ii) $-3 < x < 0$

(iii) $0 < x < 2$

(iv) $1 < x < 3$

[D] Consider the following graph of *the derivative* $f'(x)$ of a function $f(x)$. For which of the following x-values does $f(x)$ have a point of inflection?



(i) $x = -6$

(ii) $x = 1$

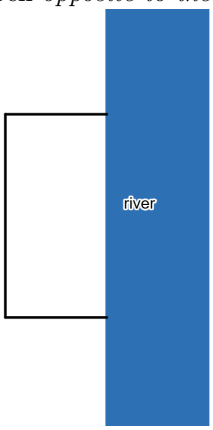
(iii) $x = 2$

(iv) $x = 3$

Optimization

14 points for scoring at least 2 out of 3.

[A] A farmer has 1800 feet of fence and wishes to fence off a rectangular pen using a long, straight river instead of fence as one side of the pen, as in the image below. What length of fence should the farmer make the side of the pen *opposite to the river* be in order to maximize the area of the pen?



(i) 30 feet

(ii) $\sqrt{1800}$ feet

(iii) 60 feet

(iv) 450 feet

(v) 900 feet

[B] Suppose $f(x)$ is a differentiable function on a closed, bounded interval $[a, b]$. Which x-values *must* be checked if we wish to definitively find the absolute extrema of $f(x)$ on $[a, b]$?

(I) The endpoints $x = a$ and $x = b$

(II) Values c in $[a, b]$ so that $f(c) = 0$.

(III) Values c in $[a, b]$ so that $f'(c) = 0$.

(IV) Values c in $[a, b]$ so that $f'(c)$ is undefined but $f(c)$ is defined.

(V) Values c in $[a, b]$ so that $f''(c) = 0$.

(VI) Values c in $[a, b]$ so that $f''(c)$ is undefined but $f'(c)$ is defined.

(i) (II) only

(ii) (III) only

(iii) (V) only

(iv) (I) and (II)

(v) (I), (III), and (IV)

(vi) (I), (V), and (VI)

(vii) (I), (III), (IV), (V), and (VI)

[C] A box with volume 10 cubic feet is to be constructed with a square base of side length x feet. Materials for the base of the box cost 4 dollars per square foot, for the sides of the box cost 2 dollars per square foot, and for the lid of the box cost 3 dollars per square foot. Which of the following represents the cost of the box $C(x)$ as a function of x?

(i) $C(x) = \frac{70}{x} + 8\sqrt{10x}$

(ii) $C(x) = 2x^2 + \frac{40}{x}$

(iii) $C(x) = 6x^2 + \frac{120}{x}$

(iv) $C(x) = 7x^2 + \frac{80}{x}$

(v) $C(x) = \frac{5}{2}x - \frac{1}{2}x^3$

(vi) $C(x) = \frac{5}{4}x - \frac{7}{8}x^3$

(vii) $C(x) = \frac{15}{2}x - \frac{3}{2}x^3$

Other Applications of Derivatives

14 points for scoring at least 2 out of 4.

[A] What is the value of the limit $\lim_{x \to \pi} \dfrac{x^2 - \pi^2}{\sin(x)}$?

(i) $-\infty$

(ii) -2π

(iii) 0

(iv) 2π

(v) $+\infty$

(vi) does not exist

[B] Which of the following is the approximation of $e^{0.1}$ which is obtained by using the linearization to $f(x) = e^x$ at $x = 0$?

(i) 0.1

(ii) 1

(iii) 1.1

(iv) $e^{0.1}$

[C] What is the slope of the tangent line to the graph given by the equation $x^2 + xy + y^2 = 3$ at the point $(1, 1)$?

(i) -5

(ii) -2

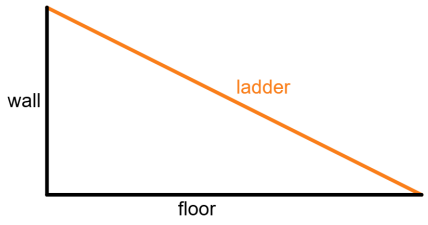
(iii) $-\frac{2}{3}$

(iv) -1

(v) 0

(vi) undefined

[D] A 5-foot ladder slides down a wall at a rate of 1 foot per minute. Below is a diagram showing an arbitrary time between when the ladder starts and stops sliding down the wall. At what rate and in which direction is the bottom of the ladder sliding when the top of the ladder is 3 feet above the ground?



(i) The base of the ladder is sliding **away** from the wall at a rate of $\frac{3}{4}$ feet per minute.

(ii) The base of the ladder is sliding **toward** from the wall at a rate of $\frac{3}{4}$ feet per minute.

(iii) The base of the ladder is sliding **away** from the wall at a rate of 4 feet per minute.

(iv) The base of the ladder is sliding **toward** from the wall at a rate of 4 feet per minute.

(v) The base of the ladder is sliding **away** from the wall at a rate of $\sqrt{24}$ feet per minute.

(vi) The base of the ladder is sliding **toward** from the wall at a rate of $\sqrt{24}$ feet per minute.

Antiderivatives and Integrals

6 points for scoring at least 3 out of 4.

[A] Which of the following is an antiderivative of $\sqrt[3]{x^2}$?

(i) $\frac{2}{3}x^{-1/3}$

(ii) $\sqrt[3]{2x}$

(iii) $\frac{x}{\sqrt[3]{3}}$

(iv) $x^{2/3}$

(v) $x^{3/2}$

(vi) $\frac{3}{5}x^{5/3}$

(vii) $\frac{3}{2}x^{5/3}$

[B] Suppose $F(x)$ and $G(x)$ are antiderivatives of the same function. How are the graphs of $F(x)$ and $G(x)$ related?

(i) The graph of $F(x)$ must be exactly the same as the graph of $G(x)$.

(ii) The graph of $F(x)$ can be obtained from the graph of $G(x)$ by shifting horizontally, if they aren't the same.

(iii) The graph of $F(x)$ can be obtained from the graph of $G(x)$ by shifting vertically, if they aren't the same.

(iv) The graph of $F(x)$ can be obtained from the graph of $G(x)$ by stretching/compressing horizontally, if they aren't the same.

(v) The graph of $F(x)$ can be obtained from the graph of $G(x)$ by stretching/compressing vertically, if they aren't the same.

(vi) The graphs of $F(x)$ and $G(x)$ might not have any meaningful relation.

[C] Which of the following is an antiderivative of $\sin(x) + \cos(x) + e^x$?

(i) $\cos(x) - \sin(x) + e^x$

(ii) $-\cos(x) + \sin(x) + e^x$

(iii) $\cos(x) - \sin(x) + xe^{x-1}$

(iv) $-\cos(x) + \sin(x) + xe^{x-1}$

(v) $\cos(x) - \sin(x) + \frac{e^{x+1}}{x+1}$

(vi) $-\cos(x) + \sin(x) + \frac{e^{x+1}}{x+1}$

[D] What is the net signed area between $y = x^3 - x^2$ and the x-axis from $x = 1$ to $x = 2$?

(i) -7

(ii) $-\frac{1}{12}$

(iii) $\frac{17}{12}$

(iv) 4

(v) 7

(vi) 8